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# Heat transfer in particulate flows

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**Abstract**—A phenomenological model is developed for particulate flows in pipes. The gas–solid mixture is modeled as a variable density, variable heat capacity fluid with the solid phase contributing to fluctuations in the mean properties of the flow. The momentum and energy equations are developed for the steady-state flow of such a fluid, and when averaged, several new terms appear that emanate from the fluctuation products. Simple diffusivity theory is used to obtain closure equations for the last terms. Finally, the resulting ordinary differential equations are solved and results are obtained for the heat transfer coefficient of the gas–solid mixtures.

## INTRODUCTION

PARTICULATE flows have been used since the 1920s for the transportation of solid materials. The subject of heat transfer in particulate flows came into scientific prominence during the 1950s when seeding the flow with solids was considered as a heat transfer augmentation technique. During that time, experimental work by Farbar and co-workers [1, 2], Dansinger [3], and Tien and Quan [4, 5] established a data basis and experimental correlations for the heat transfer coefficients of air–solid mixtures. A review of the subject [6] gives an account of the major projects undertaken before 1966. Since then a comprehensive review by Depew and Kramer [7], and papers by Briller and Peskin [8] and Shrayber [9], have added to the scientific knowledge on the subject. Numerical studies [10] provide alternative methods to obtaining engineering results.

The attempts to use solid particles in order to augment the heat transfer in fluids were abandoned, because of erosion and cleaning problems. However, the subject of heat transfer in particulates is still of great interest in pneumatic conveying applications, drying of solids [11], as an approximation to the heat transfer in mist flows [12] and also in fluidized bed applications.

The objective of this work is to develop and test an analytical model, which would predict the heat transfer characteristics of particulate flows from low to intermediate loadings (up to $m^* = 10$). The model is based on the phenomenological approach, which treats the flowing mixture as a turbulent, single-phase fluid of variable density and heat capacity. The substance flows in a circular duct and exhibits variable local density and heat capacity.

It is known from experiments that in vertical pipes the concentration of the solids is symmetric and that it is well approximated by a parabolic curve [13, 14]. The pressure acts in the longitudinal direction and the pressure gradient in the radial direction is zero. The variation of local density with the velocity gradient gives rise to an extra term in the Reynolds stresses ($\bar{\rho}\,\overline{v'u}$), which contributes to the shear stress of the flow [15, 16]. Similarly the density and heat capacity variation in a cross-section of the flow contribute certain terms in the heat transfer equation; these terms are accounted for here. Appropriate closure equations are obtained for the added terms in accordance with diffusivity theory. The resulting expressions for the momentum and heat transfer are solved numerically to yield the velocity and temperature profiles, the shear stress and the heat transfer at the wall.

The same approach to modeling particulate flows has been tested before with good results for gas–solid systems [16] and liquid–solid systems [17]. Here, the approach is extended to include the heat transfer of suspensions. As a phenomenological model it yields accurate results for the time-average quantities such as velocity, density and temperature profiles, friction factors and heat transfer coefficients. However, it does not reveal the behavior of individual particles and does not answer any questions about particle interactions. It is a simple mechanistic model that would be of value to engineers and to designers of pneumatic conveying systems.

## 2. PROBLEM FORMULATION

This study examines the steady-state flow and heat transfer of a gas–solid mixture with constant heat flux. The flow is taken to be axisymmetric. This assumption is valid in vertical and horizontal flows of high Froude numbers based on the shear velocity of the flow and particle diameter ($Fr^* = V^*/\sqrt{gd}$). Thus, horizontal flows of particles where dunes are not formed and notable lack of symmetry is not observed will be represented by the proposed model.

The boundary conditions of the momentum and energy equations for pipe flows show that the problem has two dimensions $r$ and $z$. Given that steady flows are of interest, it appears that temperature and velocity gradients with respect to $z$ are much smaller than those with respect to $r$. Changes in the longitudinal direction are much slower than changes in the radial direction and, therefore, all derivatives with respect to $z$ (except of

## NOMENCLATURE

| | |
|---|---|
| $c_p$ | specific heat capacity |
| $d$ | particle diameter |
| $Fr$ | Froude number |
| $g$ | gravitational acceleration |
| $h$ | heat transfer coefficient |
| $I_1, I_2, I_3$ | integrals |
| $k$ | thermal conductivity |
| $l$ | length scale |
| $m^*$ | mass flow ratio |
| $Nu$ | Nusselt number |
| $p$ | pressure |
| $Pr$ | Prandtl number |
| $q$ | rate of heat transfer per unit area |
| $r$ | radial distance |
| $r_0$ | pipe radius |
| $Re$ | Reynolds number |
| $T$ | temperature |
| $\Delta T_B$ | bulk temperature difference |
| $u$ | longitudinal velocity |
| $v$ | transverse velocity |
| $V^*$ | shear velocity |
| $x$ | dimensionless radial distance |
| $y$ | transverse dimension |
| $y_0$ | thickness of boundary layer |
| $z$ | longitudinal dimension. |

Greek symbols

| | |
|---|---|
| $\alpha$ | area ratio |
| $\gamma$ | density parameter |
| $\delta$ | specific heat ratio |
| $\varepsilon_\phi$ | diffusivity of variable $\phi$ |
| $\mu$ | gas viscosity |
| $\rho$ | density |
| $\tau$ | shear stress. |

Subscripts

| | |
|---|---|
| G | gas |
| S | solid |
| w | wall. |

Superscripts and other symbols

| | |
|---|---|
| $\bar{}$ | time average |
| $\langle\ \rangle$ | space average |
| s | superficial |
| * | pertaining to $V^*$ |
| ′ | perturbations |
| + | dimensionless. |

pressure gradients) may be omitted in the conservation equations. This assumption has been used in aerodynamics and particulate flows with success [16]. In the absence of radial pressure gradients the resulting form of the momentum equation becomes:

$$\frac{dp}{dz} = \frac{\mu d}{r\,dr}\left(r\frac{du}{dr}\right) - \frac{1}{r}\frac{d}{dr}(r\tau') + g\rho, \tag{1}$$

which yields:

$$\frac{r}{2}\frac{dp}{dz} = \mu\frac{du}{dr} - \tau' + \frac{g}{r}\int_0^r \rho r\,dr. \tag{2}$$

The Reynolds stress $\tau'$ may be written:

$$\tau' = -(\bar{u}\,\overline{\rho' v'} + \bar{\rho}\,\overline{u'v'} + \overline{\rho' u' v'}). \tag{3}$$

The third term on the RHS of equation (3) represents a triple product of the fluctuation and may be neglected in comparison to the other terms as in refs. [16] and [17]. Thus, the shear stress at the wall may be written:

$$\tau_w\frac{r}{r_0} = \mu\frac{d\bar{u}}{dr} + \bar{u}\,\overline{\rho' v'} + \bar{\rho}\,\overline{u'v'} - g\left\{\frac{r}{r_0^2}\int_0^{r_0}\bar{\rho} r\,dr - \frac{1}{r}\int_0^r \bar{\rho} r\,dr\right\}. \tag{4}$$

The energy equation may be written in a similar way when expressed in terms of the heat flowing inside an annulus of radius $r$:

$$q = k\frac{d\bar{T}}{dr} + \bar{\rho}\,\bar{c}_p\,\overline{T'v'} + (T - T_w)\bar{\rho}\,\overline{c_p' v'} + (\bar{T} - T_w)\bar{c}_p\,\overline{\rho' v'}. \tag{5}$$

Primed terms in equations (4) and (5) represent the time fluctuations of the variables, while the terms with a bar represent the time averages. Again third-order fluctuation products are neglected.

Equation (5) shows that the heat transfered by gas–solid mixtures is augmented because of the appearance of the last two terms, which emanate from the variable density and specific heat of the mixture. These terms may be considered as the contribution of the solid particles to the heat transfer when they move randomly in the radial direction. In the present model this random motion of the particles is regarded as an added contribution to the turbulence of the single-phase variable-density fluid under consideration.

## 3. CLOSURE EQUATIONS

A glance at the resulting equations (5) and (6) shows that closure equations are needed for the expression of the time-average products $\overline{u'v'}$, $\overline{T'v'}$, $\overline{\rho' v'}$ and $\overline{c_p' v'}$. In single-phase flows several turbulence modeling techniques provide such closure equations for the average of perturbation products. Of these techniques, Prandtl's mixing-length hypothesis [18] yields accurate predictions for the heat and momentum transfer in pipes. However, terms such as $\overline{\rho' v'}$ or $\overline{c_p' v'}$ are uncommon in the incompressible flow literature. This creates the problem of deriving suitable closure equations for them. In addition, one expects that the known terms ($\overline{u'v'}$ and $\overline{T'v'}$) may need different expressions in the particulate flows for accurate predictions. The problem would have been easier if one had access to experimental data for all the perturbation products in particulate flows, but no such data are available at present. For this work it was decided to resort to expressions emanating from an eddy diffusivity hypothesis and adopt closure equations consistent with the theories of single-phase flows. Thus any fluctuation product term $\overline{v'\phi'}$, is written as:

$$\overline{v'\phi'} = -\varepsilon_\phi \frac{\mathrm{d}\bar{\phi}}{\mathrm{d}y} \tag{6}$$

where $\varepsilon_\phi$ is the diffusivity of the quantity $\phi$. Subsequently the diffusivity $\varepsilon_\phi$ is approximated by two length scales $l_u$ and $l_\phi$ (similar to the mixing length) and the gradient of $\bar{u}$:

$$\varepsilon_\phi = l_u l_\phi \left|\frac{\mathrm{d}\bar{u}}{\mathrm{d}y}\right|. \tag{7}$$

Equations (6) and (7) yield upon substitution of $y = r_0 - r$:

$$\overline{v'\phi'} = -l_u l_\phi \frac{\mathrm{d}\bar{\phi}}{\mathrm{d}r}\frac{\mathrm{d}\bar{u}}{\mathrm{d}r}. \tag{8}$$

Therefore, the closure equations for the unknown fluctuation products become:

$$\overline{u'v'} = -l_u^2 \frac{\mathrm{d}\bar{u}}{\mathrm{d}r}\frac{\mathrm{d}\bar{u}}{\mathrm{d}r} \tag{8a}$$

$$\overline{\rho'v'} = -l_\rho l_u \frac{\mathrm{d}\bar{\rho}}{\mathrm{d}r}\frac{\mathrm{d}\bar{u}}{\mathrm{d}r} \tag{8b}$$

$$\overline{c_p'v'} = -l_{c_p} l_u \frac{\mathrm{d}\bar{c}_p}{\mathrm{d}r}\frac{\mathrm{d}\bar{u}}{\mathrm{d}r} \tag{8c}$$

$$\overline{T'v'} = -l_T l_u \frac{\mathrm{d}\bar{T}}{\mathrm{d}r}\frac{\mathrm{d}\bar{u}}{\mathrm{d}r}. \tag{8d}$$

Of the length scales in equations (8a)–(8d), $l_u$ is well known in single-phase flows where it is sometimes referred to as Prandtl's mixing length. Of the others, $l_T$ also appears in the heat transfer equations in single-phase flows. The ratio $l_u/l_T$ (also called turbulent Prandtl number, $Pr_t$) is approximately equal to 1 in tubes (Reynolds analogy) [19]. The other length scales $l_\rho$ and $l_{c_p}$ are unknown and their actual values may only be deduced from experimental results. Both of these variables appear because of the random motion of particles in the fluid. Given that they are manifestations of the same phenomenon (motion of particles) they must be equal, that is $l_\rho = l_{c_p}$. Because of lack of experimental data that would yield either of them, it is assumed here that they are both proportional to $l_u$ with the constant of proportionality being a function of the solids content:

$$l_\rho = l_{c_p} = f(m^*) l_u. \tag{9}$$

The function $f(m^*)$ is to be determined later from comparison with experimental data.

The value of $l_u$ is obtained by the Nikuradse equation [20] which yields accurate results for the single-phase pipe flows:

$$l_u = r_0[0.14 - 0.08(r/r_0)^2 - 0.06(r/r_0)^4]. \tag{10}$$

As stated above, $l_T = l_u$ and $l_\rho$ and $l_{c_p}$ are given by equation (9). Equations (6)–(10) complete the choice of closure equations for the present model.

## 4. THE DENSITY PROFILE

In all particulate flows the gas phase is continuous and the solids are dispersed. Experiments by Soo *et al.* [13, 21, 22], Spencer *et al.* [14] and Peskin [22] show that, in the absence of electrostatic effects, the solid phase will concentrate towards the center of the pipe. This is especially valid for the high-Froude-number flows considered in this work. Therefore, the time-average density $\bar{\rho}$ will be close to the gas density at the wall of the pipe and will attain its maximum value, $\rho_m$, at the center. Of course, this density $\rho_m$ and the general density distribution depend on the mass flow rate of solids in the flow. Correlations of experimental data [16, 22, 23] indicate that a suitable expression for the density distribution is:

$$\bar{\rho} = \rho_G \left[1 + \gamma \frac{r_0 - r}{r_0}\right]^m, \tag{11}$$

where $m$ is approx. 0.5 (it varies between 0.4 and 0.6) and $\gamma$ is a parameter depending on the mass flow rate of solids. The space-average density of the flow $\langle\rho\rangle$ may be obtained by integrating equation (11)

$$\langle\rho\rangle = \frac{2}{r_0^2}\int_0^{r_0} \rho r\,\mathrm{d}r = \frac{2\rho_G[(1+\gamma)^{m+2} - 1]}{\gamma^2(m+1)(m+2)} - \frac{2\rho_G}{\gamma(m+1)}. \tag{12}$$

If one is interested in the local time- or space-average volumetric percentage of solids ($\bar{\alpha}$ or $\langle\alpha\rangle$), this may be obtained from the density distribution function:

$$\bar{\alpha} = \frac{\bar{\rho} - \rho_G}{\rho_S - \rho_G} \tag{13}$$

$$\langle\alpha\rangle = \frac{\langle\rho\rangle - \rho_G}{\rho_S - \rho_G}. \tag{14}$$

From the density profile one may easily deduce the specific heat profile for the fluid under consideration. The expression for the specific heat becomes:

$$\bar{c}_p = c_{pG}\left[\delta + \frac{\rho_G}{\bar{\rho}}(1-\delta)\right] \tag{15}$$

where $\delta$ is the ratio of specific heats ($\delta = c_{ps}/c_{pG}$).

The derivatives of the density and specific heat capacity needed for the closure equations may be easily obtained from

$$\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}r} = -\frac{\rho_G \gamma m}{r_0}\left(1 + \gamma\frac{r_0 - r}{r_0}\right)^{m-1} \tag{16}$$

and

$$\frac{d\bar{c}_p}{dr} = c_{pG}\frac{\rho_G}{\bar{\rho}^2}\frac{d\bar{\rho}}{dr}(\delta-1). \tag{17}$$

## 5. MOMENTUM AND HEAT TRANSFER EQUATIONS. DIMENSIONLESS FORM

Under the stated assumptions for the closure equations the shear stress and heat transfer equations for the model become:

$$\tau_w\frac{r}{r_0} = \mu\frac{d\bar{u}}{dr} - \bar{u}f(m^*)l_u^2\frac{d\bar{\rho}}{dr}\frac{d\bar{u}}{dr} - \bar{\rho}l_u^2\frac{d\bar{u}}{dr}\frac{d\bar{u}}{dr} + g\left[\frac{r}{r_0^2}\int_0^{r_0}\bar{\rho}r\,dr - \frac{1}{r}\int_0^r\bar{\rho}r\,dr\right], \tag{18}$$

and

$$q = k\frac{d\bar{T}}{dr} - l_u^2\bar{\rho}\bar{c}_p\frac{d\bar{T}}{dr}\frac{d\bar{u}}{dr} - (\bar{T}-T_w)\bar{\rho}f(m^*)l_u^2\frac{d\bar{c}_p}{dr}\frac{d\bar{u}}{dr} - (\bar{T}-T_w)\bar{c}_pf(m^*)l_u^2\frac{d\bar{\rho}}{dr}\frac{d\bar{u}}{dr}. \tag{19}$$

For computational simplicity the shear stress and heat transfer equations above will be derived in dimensionless form. Time-average dimensionless variables appear with a '+' superscript and are defined as follows:

$$\rho^+ = \frac{\bar{\rho}}{\rho_G}, \tag{20}$$

$$c_p^+ = \frac{\bar{c}_p}{c_{pG}}, \tag{21}$$

$$u^+ = \frac{\bar{u}}{V^*} = \frac{\bar{u}}{\sqrt{-\tau_w/\rho_G}}, \tag{22}$$

($V^*$ is the shear velocity of the flow) and,

$$T^+ = (\bar{T}-T_w)\frac{c_{pG}V^*\rho_G}{q_w}. \tag{23}$$

Thus, the momentum equation written in terms of shear stress appears as follows:

$$-x = \frac{1}{Re^*}\frac{du^+}{dx} - u^+l^{2+}f(m^*)\frac{d\rho^+}{dx}\frac{du^+}{dx} - \rho^+l^{2+}\frac{du^+}{dx}\frac{du^+}{dx} - \frac{1}{Fr^{*2}}\left[x\,G(1) - \frac{1}{x}G(x)\right], \tag{24}$$

where

$$Re^* = \frac{r_0V^*\rho_G}{\mu}, \tag{24a}$$

$$Fr^* = V^*/\sqrt{gr_0}, \tag{24b}$$

$$G(x) = \int_0^x \rho^+x\,dx, \tag{24c}$$

$$x = r/r_0, \tag{24d}$$

and

$$l^+ = l_u/r_0. \tag{24e}$$

Similarly the heat transfer equation appears as follows in dimensionless form:

$$-\frac{H(x)}{H(1)} = \frac{1}{Re^*\,Pr}\frac{dT^+}{dx} - \rho^+c_p^+l^{2+}\frac{dT^+}{dx}\frac{du^+}{dx} - \rho^+T^+f(m^*)l^{2+}\frac{dc_p^+}{dx}\frac{du^+}{dx} - c_p^+T^+f(m^*)l^{2+}\frac{d\rho^+}{dx}\frac{du^+}{dx}, \tag{25}$$

where

$$H(x) = \int_0^x c_p^+\rho^+u^+x\,dx, \tag{25a}$$

and the Prandtl number has its usual significance:

$$Pr = \frac{\mu c_p}{k}. \tag{25b}$$

## 6. SOLUTION OF THE WORKING EQUATIONS

A glance at the final form of the shear stress and heat transfer equations (24) and (25) shows that they are a system of coupled non-linear ordinary differential equations in $u^+$ and $T^+$. Actually equation (24) is a quadratic equation in $du^+/dx$ alone and its algebraic solution yields the derivative of the dimensionless velocity:

$$\frac{du^+}{dx} = \frac{-B\pm\sqrt{B^2-4AC}}{2A} \tag{26}$$

where

$$A = \rho^+l^{2+}, \tag{26a}$$

$$B = -\frac{1}{Re^*} + u^+l^{2+}f(m^*)\frac{d\rho^+}{dx}, \tag{26b}$$

and

$$C = -x + \frac{1}{Fr^{*2}}\left[xG(1) - \frac{1}{x}G(x)\right]. \tag{26c}$$

Given the form of the density function and the fact that the flows examined here are of high Froude numbers ($Fr^* > 1$), $C$ and $B$ are always negative while $A$ is positive. It follows then that the above equation always has one negative root which is of interest here and is given by the following equation:

$$\frac{du^+}{dx} = \frac{-B-\sqrt{B^2-4AC}}{2A} \leqslant 0. \tag{27}$$

Integration of equation (27) yields the velocity distribution of the flow in the cylindrical pipe. The parameters $Re^*$ and $Fr^*$ are determined by iteration as will be explained in the next section.

The determination of $u^+$ and $du^+/dx$, in effect, decouples the heat transfer equation from the shear stress equation. When the dimensionless velocity and its derivatives are known, equation (25) yields a first-order non-linear ordinary differential equation for the temperature gradient $dT^+/dx$:

$$\frac{dT^+}{dx} = \frac{-\dfrac{H(x)}{H(1)} + T^+ l^{2+} f(m^*)\left(\rho^+ \dfrac{dc^+}{dx} + c_p^+ \dfrac{d\rho^+}{dx}\right)\dfrac{du^+}{dx}}{\dfrac{1}{Re^*\, Pr} - \rho^+ c_p^+ l^{2+} \dfrac{du^+}{dx}}. \tag{28}$$

One may observe in the above equation that the added terms due to the density and specific heat capacity fluctuations tend to decrease the absolute value of the temperature gradient. This means that for a given heat transfer at the wall of the pipe the resulting temperature difference is always less than that required for the pure gas flow. Then the heat transfer coefficients and the Nusselt number will be higher in particulate-laden flows.

The boundary conditions for equations (27) and (28) are derived from the fact that $u^+$ and $T^+$ are zero at the wall of the pipe ($x = 1$). Near the wall, however, there is a laminar sublayer of thickness $y_0$ where viscous forces dominate the flow. In this thin sublayer, laminar equations of motion are valid and the concentration of particles is very small. The solution of the laminar momentum and energy equations yields the following expressions for the dimensionless velocity and temperature in the sublayer:

$$u^+ = 1 - x, \quad 1 > x > 1 - y_0^+, \tag{29}$$

and

$$T^+ = Pr(1-x), \quad 1 > x > 1 - y_0^+. \tag{30}$$

Therefore, the boundary conditions for equations (27) and (28) are:

$$u^+(y_0^+) = y_0^+, \tag{29a}$$

and

$$T^+(y_0^+) = y_0^+ Pr. \tag{30a}$$

The integration of the differential equations as supplemented by the above boundary conditions is accomplished by the Runge–Kutta method. The velocity profiles may be compared with experimental data from [7]. This comparison is shown in Fig. 1; it is evident that there is very good agreement between the data and the results obtained from the present model. Temperature profiles as derived in the present study are shown in Fig. 2. There are no available experimental data on temperature profiles to be compared.

Kramer [24] deduced from experimental data the values for the suspension eddy diffusivity $\varepsilon_m$. Some of his data for particles of $62\mu$ are compared to values of eddy diffusivity obtained from this model in Fig. 3. The abscissa in the last graph is the dimensionless form of the eddy diffusivity:

$$\varepsilon_m^+ = \frac{\varepsilon_m}{r_0 V^*} = \frac{\tau}{r_0 V^* \bar{\rho}\, d\bar{u}/dr}. \tag{31}$$

Again it may be seen that there is good agreement of data and results from this model.

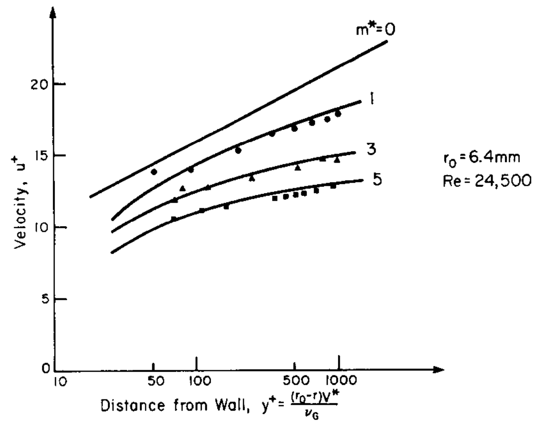


FIG. 1. Comparison of velocity profiles with data from ref. [7].

## 7. AVERAGE QUANTITIES OF INTEREST

In the case of pipe flows, the space-average velocity, mass flux and bulk temperature difference are quantities of practical interest:

$$\langle u \rangle = \frac{2}{r_0^2}\int_0^{r_0} \bar{u} r\, dr = V^* I_1, \tag{32}$$

$$\langle G \rangle = \frac{2}{r_0^2}\int_0^{r_0} \bar{\rho}\bar{u} r\, dr = V^* \rho_G I_2 \tag{33}$$

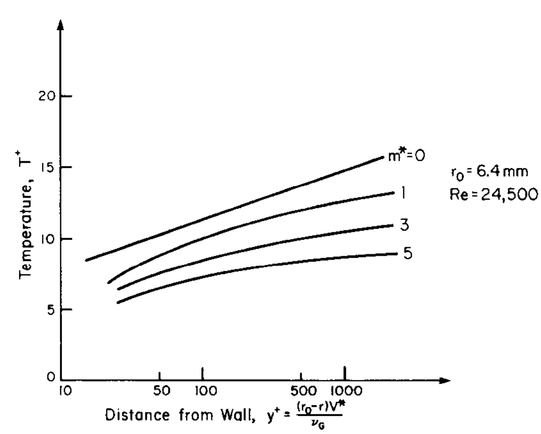


FIG. 2. Temperature profiles for several loadings.

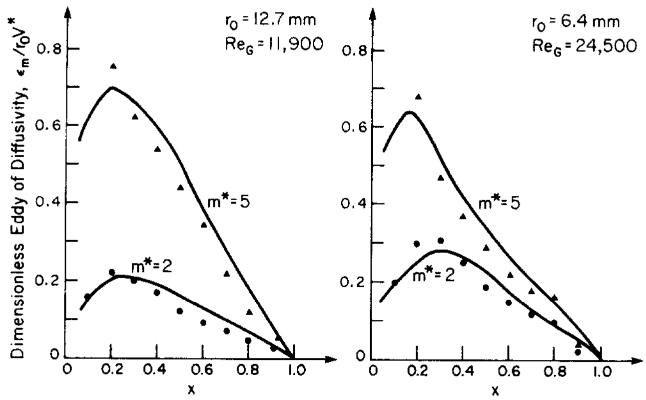


FIG. 3. Eddy diffusivity comparisons with data from ref. [24].

and

$$\Delta T_B = \frac{2}{\rho_G c_{pG} V^* r_0^2} \int_0^{r_0} \bar{\rho}\bar{c}_p \bar{u}(T_w - \bar{T}) r \, dr$$

$$= \frac{q_w I_3}{\rho_G c_{pG} V^*}. \quad (34)$$

The integrals $I_1$, $I_2$, and $I_3$, are dimensionless and may be evaluated from the known dimensionless velocity and temperature distributions. In terms of the above space-average quantities, the superficial velocities of the solids and gases may be written as follows [16, 25]:

$$V_G^s = V^* \frac{\rho_S I_1 - \rho_G I_2}{\rho_S - \rho_G}, \quad (35)$$

and

$$V_S^s = V^* \rho_G \frac{(I_2 - I_1)}{\rho_S - \rho_G}. \quad (36)$$

The ratio of mass flow rates $m^*$ is:

$$m^* = \rho_S \frac{I_2 - I_1}{\rho_S I_1 - \rho_G I_2}. \quad (37)$$

Finally, the heat transfer coefficient for the flow, $h$ is:

$$h = \frac{q_w}{\Delta T_B} = \frac{\rho_G c_{pG} V^*}{I_3}, \quad (38)$$

and the Nusselt number becomes:

$$Nu = \frac{2hr_0}{k} = 2\frac{Re^* Pr}{I_3}. \quad (39)$$

In a flowing gas–solid mixture, the ratio of the mass flow rates $m^*$, the dimension $r_0$ and the superficial gas velocity $V_G^s$ are known, as well as the thermodynamic properties of gases and solids. In general, $\gamma$ (for the density profile) and $V^*$ are unknown. The solution for the set of equations (35), (37) and (39) is achieved by iteration according to the following procedure:

(a) Values for $V^*$ and $\gamma$ are assumed.
(b) $Fr^*$, $Re^*$ are calculated according to the above values.
(c) Equations (27), (28), (32)–(34) are integrated simultaneously by the Runge–Kutta method to yield the dimensionless velocity and temperature distributions as well as the integrals $I_1$, $I_2$ and $I_3$.
(d) A check is performed to see if equations (35) and (36) are satisfied for the known values of $V_G^s$ and $m^*$. If not, $V^*$ and $\delta$ are modified and steps (b)–(d) are repeated until agreement of 1% for the values of $V_G^s$ and $m^*$ is achieved.

Convergence of the solutions happens very quickly and in general the whole iterative procedure takes less than 20 s CPU time on a VAX 11/780 system. It appears, therefore, that the evaluation of the heat transfer coefficient by this method is a very economical way to solving the conservation equations for the two-phase mixture.

Regarding the function $f(m^*)$, its final expression evolved from comparison of the results obtained with the correlation $B$ derived by Pfeffer *et al.* [6].

$$\frac{h}{h_G} = 1 + 4 \cdot Re_G^{-0.32} \delta m^* \quad (40)$$

The comparison showed that $f(m^*)$ may be given as a quadratic function of $m^*$:

$$f(m^*) = 0.031 - 0.070m^* + 0.045m^{*2}. \quad (41)$$

## 8. COMPARISON WITH EXPERIMENTAL DATA AND CORRELATIONS

As indicated before, there are several known experimental studies on the subject of heat transfer in particulate flows [1–4, 7, 26–28]. Due to the large number of parameters involved, the effects of radiation and other experimental uncertainties, some of the studies appear to be less reliable than others [6]. It is commonly agreed, however, that the most important parameters are $Re_G$, $Pr$, $m^*$ and $\delta$ (sometimes the latter two appear as the product $m^*\delta$). Particle size does not enter the correlations; it appears in fact that it plays a minor role in the total heat transfer coefficient as indicated in Pfeffer *et al.*'s survey [6].

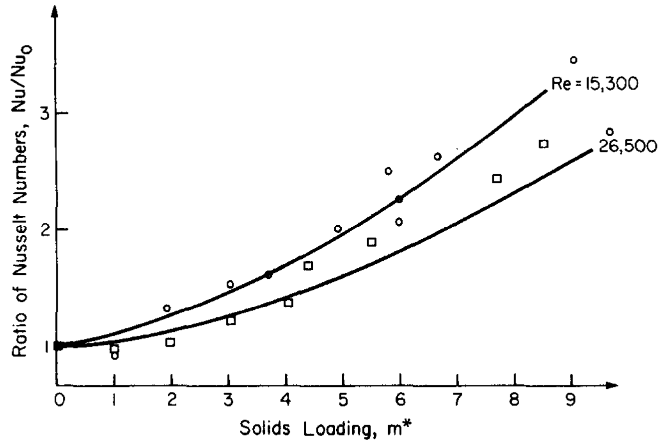


FIG. 4. Comparison of Nusselt numbers with data from ref. [2].

It is common practice to present in data or correlations the ratio of Nusselt numbers for particle flow divided by that of gas-only flow:

$$\frac{Nu}{Nu_0} = \frac{Nu(Re_G, Pr, m^*, \delta)}{0.023 Re_G^{0.8} Pr^{0.4}}. \tag{42}$$

The results obtained by the method described in this paper are compared with experimental data and correlations. Figure 4 shows comparisons with experimental data from Farbar and Depew [2]. The same type of comparison is made in Fig. 5 with data from Dansinger [3]. It may be seen that there is good agreement of individual data sources and the results of the present method.

Next, comparisons are made with correlations derived from experimental data. As such, the correlations evolving from the works of Farbar and Morley [1], Wachtel *et al.* [28] and Pfeffer *et al.* [6] (correlation B) were chosen. The last one is derived from several sets of experimental data and represents a 'middle of the road' correlation. The results of this comparison are shown in Fig. 6 as ratio of Nusselt numbers, vs $m^*\delta$, and in Fig. 7 vs Reynolds number. Again it is observed that the method presented here agrees reasonably well with the experimental correlations. Here it must be emphasized that the good fit with the Pfeffer *et al.* correlation [6] is due to the choice of the function $f(m^*)$. However, the results are not very sensitive on the choice of the last function; fairly good agreement with data and correlations is obtained even if this function has the value of unity.

It may be observed in Fig. 7 that the present results approach the value 1 at large Reynolds numbers in agreement with data from ref. [8].

The present model is purely a phenomenological one; it yields useful results about the space-average flow coefficients but it does not answer any questions pertaining to the interactions of particles and the complex phenomona associated with exchange mechanisms between the two phases. The subject of air–solid mixtures is a complex one and a detailed description of the flow field would involve a large number of variables. It is not within the scope of this paper to include a discussion of all these variables. At this time it suffices that the results of this study may be

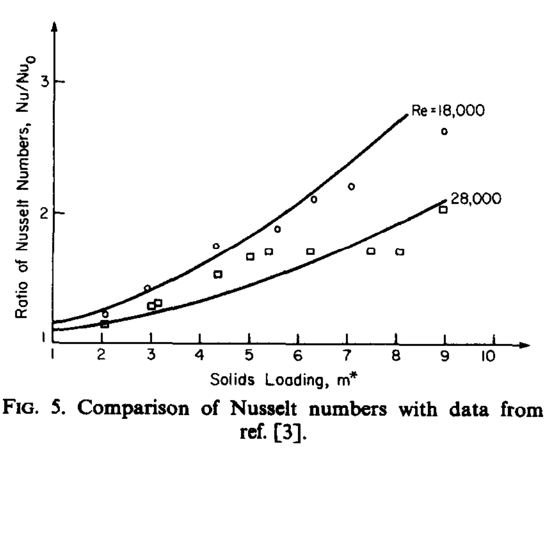


FIG. 5. Comparison of Nusselt numbers with data from ref. [3].

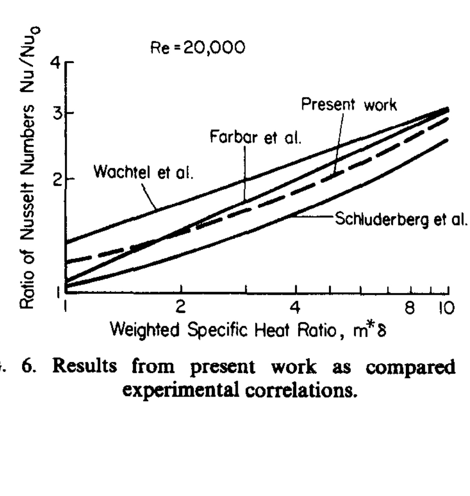


FIG. 6. Results from present work as compared with experimental correlations.

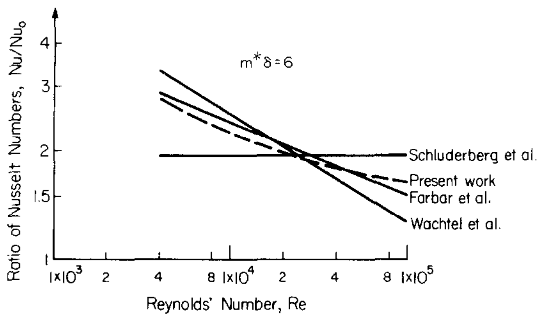


FIG. 7. Comparisons of results from present work with experimental correlations.

obtained in a simple way and appear to be accurate enough despite the mechanistic approach to the modeling of two-phase systems.

## 9. CONCLUSIONS

A model is developed for gas–solid flows based on a phenomenological approach. The presence of particles is taken to contribute to the flow perturbations, thus adding fluctuation terms to the density, temperature and specific heat capacity. When the convervation equations for such a fluid are averaged, several additional terms appear in the momentum and energy equations. These terms tend to increase the friction factor and heat transfer coefficient of particulate flows. Closure equations similar to the eddy diffusivity expressions are used to model these additional terms. Certain parameters such as mixing lengths are either determined from their values in single-phase flow or from an optimization procedure. These mixing lengths should better be inferred from experimental data.

When the energy and momentum equations are solved one may easily obtain Nusselt numbers, average velocities and friction factors as functions of the mass flow ratios, Reynolds numbers and ratios of properties. The results obtained show good agreement with other experimental data and correlations.

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## TRANSFERT THERMIQUE DANS LES ECOULEMENTS AVEC PARTICULES

**Résumé**—Un modèle phénoménologique est construit pour des écoulements à particules dans les tubes. Le mélange gaz–solide est représenté comme étant à densité variable, à chaleur massique variable avec la phase solide contribuant aux fluctuactions dans les propriétés moyennes de l'écoulement. Les équations de quantité de mouvement et d'énergie sont établies pour un écoulement permanent et en faisant la moyenne, quelques nouveaux termes apparaissent émanant des produits fluctuants. La simple théorie de diffusivité est utilisée pour obtenir les équations de fermeture pour les derniers termes. Finalement les équations différentielles sont résolues et les résultats sont obtenus pour le coefficient de transfert du mélange gaz–solide.

## WÄRMEÜBERGANG IN TEILCHENBELADENEN STRÖMUNGEN

**Zusammenfassung**—Ein phänomenologisches Modell für teilchenbeladene Strömung in Rohren wird entwickelt. Das Gas-Feststoff-Gemisch wird modelliert als Fluid mit variabler Dichte und variabler Wärmekapazität, bei dem die feste Phase Abweichungen von den mittleren Eigenschaften der Strömung verursacht. Die Impuls- und Energiegleichungen werden für die stationäre Strömung eines solchen Fluids entwickelt, wobei bei der Mittelung einige neue Terme aufgrund der Schwankungswerte auftreten. Eine einfache Diffusionstheorie wird angewandt, um für die zuletzt genannten Terme geschlossene Gleichungen zu erhalten. Schließlich werden die resultierenden gewöhnlichen Differentialgleichungen gelöst, und es werden Ergebnisse für den Wärmeübergangskoeffizienten der Gas-Feststoff-Gemische erhalten.

## ТЕПЛОПЕРЕНОС В ПОТОКАХ МАКРОЧАСТИЦ

**Аннотация**—Разработана феноменологическая модель для потоков макрочастиц в трубах. Смесь газ-твердые частицы рассматривается как жидкость с переменными плотностью и теплоемкостью с учетом вклада твердой фазы во флуктуации средних свойств потока. Для стационарного течения такой жидкости выведены уравнения импульса и энергии, как результат осреднения появляется несколько новых членов. При выводе замкнутых уравнений для последних членов используется простая аппроксимация. Получены и решены обыкновенные дифференциальные уравнения и определены в итоге коэффициенты теплообмена смесей газ-твердые частицы.